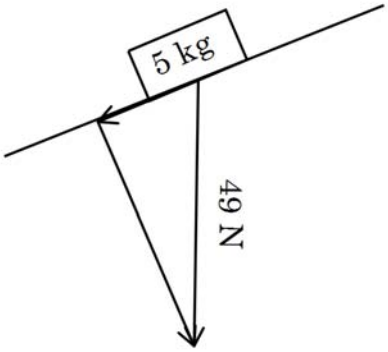
Activity 13 Projection

Aim: Construct and measure the projection of one vector onto another. Resolve vectors into components.

1. The diagram shows a 5 kg block sitting on an inclined plane parallel to the vector [7,2].



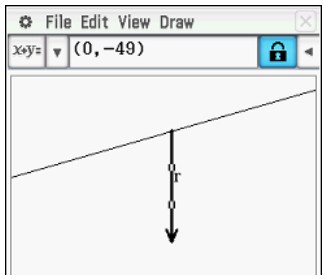
The block will remain stationary unless the component of gravitational force (49 N) that is parallel to the ramp exceeds the static friction of 14 N.

Draw inclined plane

- Open a new Geometry page
- Draw a line and constrain gradient to $\frac{2}{7}$

Draw gravitational force

- Draw vector **r** and constrain components to [0,–49]



Construct perpendicular

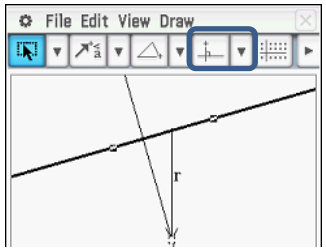
- Select the line and the point at the head of vector **r**
- Select [Draw | Construct | Perpendicular] or tap ⊥ from the Draw pull-down menu

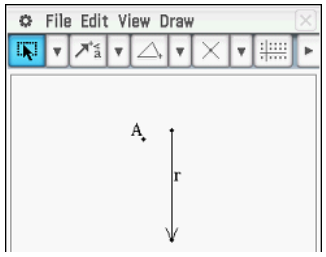
Locate point of intersection

- Select the two lines
- Select [Draw | Construct | Intersection] or tap ✕ from the Draw pull-down menu

Hide the lines

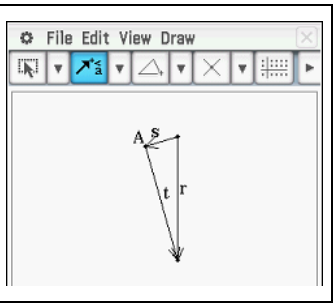
- Select the lines
- Select [Edit | Properties | Hide]





Complete vector diagram

- Draw a vector from the tail of **r** to the point of intersection
- Draw a vector from the point of intersection to the head of **r**

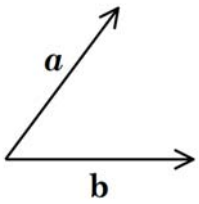


a) Determine the component of the gravitational force that is parallel to the inclined plane (vector **s** in the diagram above). This represents the **vector projection** of [0, –49] onto [7, 2].

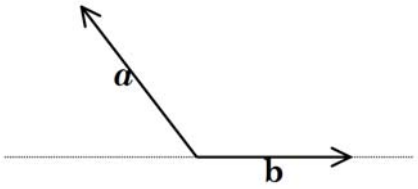
b) Determine the magnitude of the force in a). Will the block slide down the plane?

2. In each of the following diagrams draw the vector projection of **a** onto **b**.

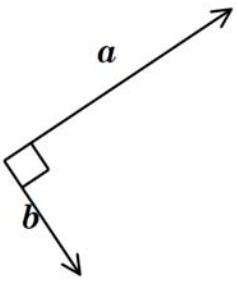
a)



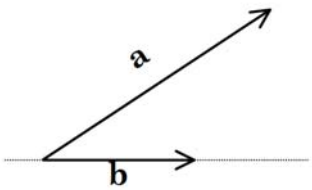
b)



c)



d)



3. Given vectors **a** = [10, –3] and **b** = [6, 1], construct the projection of **b** on **a**.

   Detailed instructions for the construction can be found in the learning notes.

   a) What is the scalar projection of **b** on **a**?

   b) What is the vector projection of **b** on **a**?

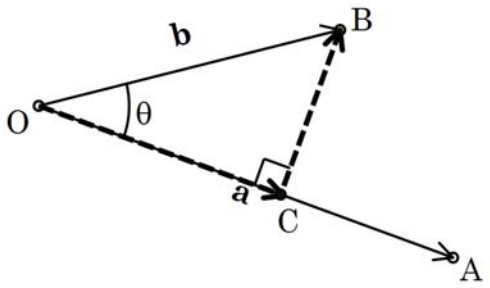
   c) What is the component of **b** perpendicular to **a**?

   d) What is the unit vector in the direction of **a**?

   e) Calculate the scalar projection of **b** on **a** multiplied by the unit vector in the direction of **a**. Compare this to your answer for b).

4.

a) Write an expression for the length of $\overrightarrow{OC}$ using θ and the length of **b**.



b) Write an expression for the length of $\overrightarrow{OC}$ using the scalar product of vectors **a** and **b**.

c) Use this result to calculate the scalar projection of:

(i) [5, 4] on [3, 0]

(ii) [–3, 5] on [3, 0]

(iii) [5, 4] on [4, –5]

(iv) [4, 0] on [2, –3].

Learning notes

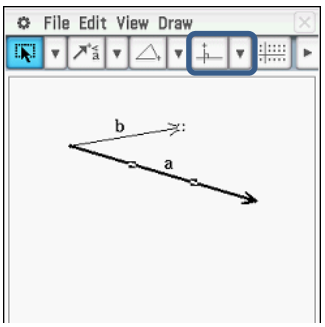
Q3 construction

Draw vectors a and b

- Open a new Geometry page
- Draw vector **a** and constrain to [10, –3]
- Draw vector **b** and constrain to [6, 1]

Draw perpendicular

- Select vector **a** and the point at the head of vector **b**
- Select [Draw | Construct | Perpendicular] or tap from the Draw pull-down menu



Locate point of intersection

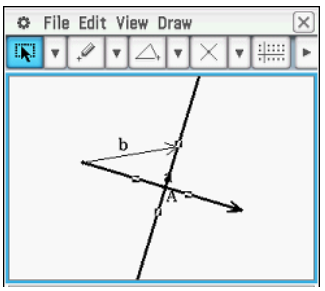
- Select **a** and the perpendicular line
- Select [Draw | Construct | Intersection] or tap from the Draw pull-down menu

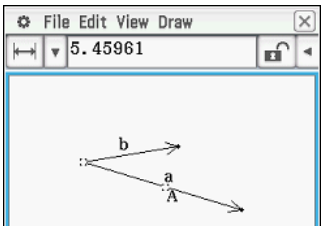
Hide the perpendicular

- Select the perpendicular
- Select [Edit | Properties |Hide]

Measure the length of the projection

- Select the tail of vector **r** and the intersection point
- Read the length from the measurement box





Draw parallel component

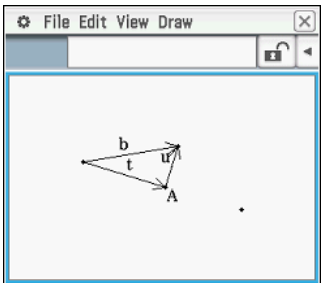
- Tap on
- Tap on tail of **a**
- Tap on intersection point

Draw perpendicular component

- Tap on intersection point and tap on head of **b**

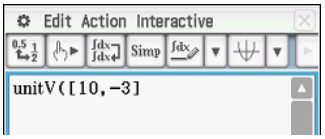
Hide vector a (needed to be able to select the component)

- Select **a**
- Select [Edit | Properties | Hide]



A unit vector is a vector 1 unit long. A unit vector in the direction of **a** is $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$

Select [Action |Vector | unitV] in Main to calculate a unit vector.



The scalar projection of a vector **a** onto another vector **b** is given by $proj_{\mathbf{a}\text{ on }\mathbf{b}} = \dfrac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{b}|}$. A positive indicates the projection is in the direction of **b**, and a negative indicates the projection is in the opposite direction to **b**.

An alternative definition of scalar projection of a vector is the length of the vector projection.

Q3 b) Use the vector definition of the dot product $\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ and compare to your answer in part a).